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A STUDY OF THE TRANSIENT BEHAVIOR OF FUEL DROPLETS DURING COMBUSTION: THEORETICAL CONSIDERATIONS FOR AERODYNAMIC STRIPPING

Prepared by

University of Illinois
Aeronautical and Astronautical Engineering Department
Urbana, Illinois

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Even though the steady or quasi-steady combustion of isolated droplets or droplet sprays has been studied extensively and is reasonably well understood, the problem of transient gas dynamics coupled to droplet combustion is so poorly understood at the present time that adequate models to describe the phenomena have yet to be proposed. The problem is, however, extremely important to the technology of combustion devices. Aside from the detonation of spray droplet mixtures, in which droplet breakup and combustion are occurring under extremely transient conditions, a full descriptive theory of this phenomena will be invaluable in furthering our understanding of such varied phenomena as liquid rocket engine instabilities, engine thrust transients, combustion in liquid propellant gun systems, high pressure fuel-air gun systems, spray ignition mechanisms, and hybrid rocket dynamics.

This report presents a theoretical analysis of one important phase of heterogeneous flow, namely a model for the aerodynamic stripping of fuel from the assumed spherical droplets.

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A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

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LIST OF SYMBOLS

Γ	density, viscosity ratio ($\Gamma = \rho_g \sqrt{\nu_g} / \rho_\ell \sqrt{\nu_\ell}$)
f	nondimensional Blasius stream function (f = G or = L)
G	nondimensional stream function for gas boundary layer Eq. (10)
K(x)	proportionality factor in spallation boundary condition, Eq. (6)
L	nondimensional stream function for Liquid Boundary Layer
l	characteristic length of flow transformation, Eq. (7)
P	pressure
r	radius, specifying contour of body of revolution
R	droplet radius
u	boundary - layer velocity along the x co-ordinate
U	potential free-stream velocity which is a function of x
U_∞	constant flow velocity far ahead of the droplet
V	boundary-layer velocity along the y co-ordinate
X	curvilinear co-ordinate measured along a meridran from the stagnation point
y	curvilinear co-ordinate at right angles to the surface
β	nondimensional Faulkner-Skan wedge parameter, Eq. (17)
ψ	ψ (x,y) stream function, Eq. (7)
ξ	nondimensional x co-ordinate, Eq. (8)
η	nondimensional y co-ordinate, Eq. (9)
μ	viscosity
ν	fluid kinematic viscosity = μ/ρ
ρ	fluid density

<u>Subscripts</u>

g	gas
l	liquid
∞	free stream
o	interface between gas-liquid boundary layer

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1.0 ABSTRACT

Even though the steady or quasi-steady combustion of isolated droplets or droplet sprays has been studied extensively and is reasonably well understood, the problem of transient gas dynamics coupled to droplet combustion is so poorly understood at the present time that adequate models to describe the phenomena have yet to be proposed. The problem is, however, extremely important to the technology of combustion devices. Aside from the detonation of spray droplet mixtures, in which droplet breakup and combustion are occurring under extremely transient conditions, a full descriptive theory of this phenomena will be invaluable in furthering our understanding of such varied phenomena as liquid rocket engine instabilities, engine thrust transients, combustion in liquid propellant gun systems, high pressure fuel-air gun systems, spray ignition mechanisms, and hybrid rocket dynamics.

This report presents a theoretical analysis of one important phase of heterogeneous flow, namely a model for the aerodynamic stripping of fuel from the assumed spherical droplets.

2.0 INTRODUCTION

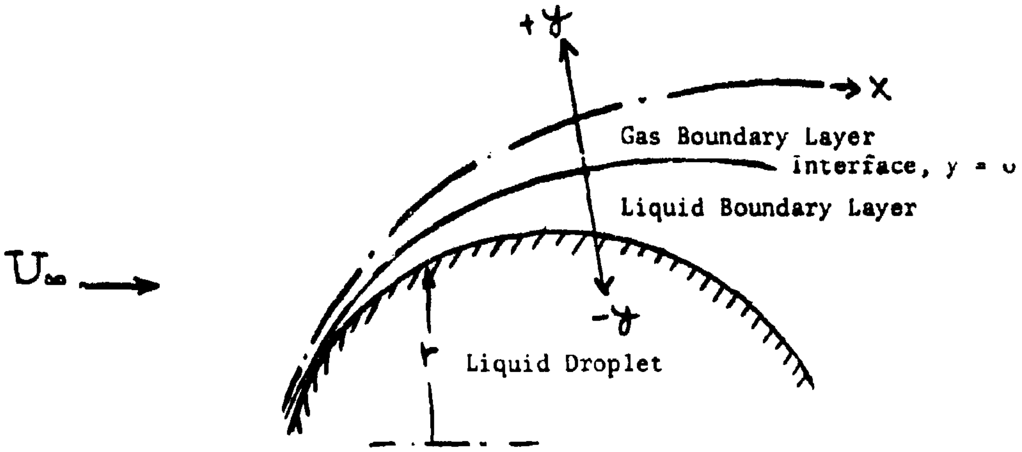
One of the most difficult problems encountered in the development of high-speed compression-ignition chambers has been the proper atomization and distribution of the fuel in the combustion chamber during the extremely short time available. Rapid combustion of the fuel does not take place as soon as it enters the combustion chamber, but a certain time, known as the ignition lag, elapses during which the temperature of the fuel is raised to its auto-ignition point by the absorption of heat from the compressed air. The rate of heat absorption by a fuel drop is directly proportional to its surface area; the rate of its temperature rise is inversely proportional to its volume. Because the surface area varies as the square of the diameter, whereas the volume varies as its cube, a small drop will have a shorter ignition lag than a larger one. The time required for the complete combustion of small drops is also less than that for the large ones; therefore, the smallest drops are the most desirable if they can be obtained without sacrificing good distribution.

This report discusses a description of one of the physical phenomena for reducing the size of the droplets, namely that of aerodynamic stripping.

3.0 DESCRIPTION OF THE PROBLEM

As a result of droplet breakup, it is usually observed that burning rates are obtainable which are higher than those possible under the conditions of low velocity and forced convection where no disintegration occurs. In order to increase the rate of a complete combustion, the stripping effects of the breaking up process of a fuel droplet is considered in this study. This theoretical study on stripping effects of a droplet in high speed air is made in order to investigate how to properly model such a physical phenomena. Therefore, the formulation of the problem, i.e., the governing equations and the proper boundary conditions are given considerable detail.

Basically, this is a two-phase viscous flow problem. Since the droplet is in a high speed air stream, the problem is further simplified by application of standard boundary layer theory. This means that a high Reynolds number flow problem is reduced to a two-phase boundary layer flow problem. In order to illustrate this problem, a sketch of the flow field is presented in the following sketch:



The two phase boundary layer flow problem has been developed for many engineering applications, such as ablation, spallation, aerodynamic shattering, etc. Ranger and Nicholls[1] studied the aerodynamic shattering of a liquid droplet. Their work gives experimental results and has attempted a theoretical description. The theoretical analysis in their study is to integrate the boundary layer equation by Karman's momentum integral method. Then an approximate solution of the velocity distribution of the gas and the liquid is posed. This is exactly the same as G. I. Taylor's analysis[2] in a somewhat different problem. With the assumption of knowing a-priori the velocity distribution, they calculated the mass stripped away from the droplet. G. W. Sutton[3] studied a related problem of hydrodynamics and heat conduction of a melting surface near the stagnation point of a high speed stream. He carefully studied the boundary conditions and formulated the problem as an axially symmetric Falkner-Skan flow. He also applied his analysis to experiments on the ablation of reinforced plastics in supersonic flow[4]. E. W. Adams investigated transient and quasi-steady performance of melting type re-entry shield[5]. There are many others who also contributed in this field; the references are included.

4.0 ASSUMPTIONS REQUIRED

Because this problem is quite complicated, some assumptions have to be made in order to simplify the problem, so that it can be made mathematically tractable. The assumptions made are as follows:

1. The flow is considered as incompressible.
2. It is assumed that the problem is one of a steady state flow.
3. There is no slip at the interface of gas and liquid boundary layers (at $y = 0$).
4. The outer solution of velocity over the spherical droplet is assumed to be $U(x) = \frac{3}{2} U_\infty \sin \frac{x}{R}$, which is the potential flow, where R is the radius of the droplet, and x is the curvilinear streamline coordinate.
5. The boundary condition at the gas liquid interface was suggested by G. I. Taylor and is $\mu_g \frac{\partial u_g}{\partial y} = \mu_\ell \frac{\partial u_\ell}{\partial y}$, since there is no slipping (see Assumption 3).
6. The centrifugal pressure gradient is small; thus $\frac{\partial P}{\partial y} \cong 0$.

It should be pointed out that the boundary layer equations, when transformed from the x, y co-ordinate system to the ξ and η system (where ξ is related to x and η is the similarity variable) have neglected the partial derivative terms with respect to ξ. Similar solutions are assumed thereby, although strictly not allowed, since the outer velocity distribution over the sphere is not of the correct power-law form. It is assumed as is usual that the non-similar terms do not appreciably modify the results.

5.0 FORMULATION OF THE PROBLEM

Since the assumptions made correspond to reasonable physical circumstances, the mathematical formulation for this problem is a straight forward application of boundary layer analysis. A difficulty that arises is to determine what boundary conditions to apply at the liquid-liquid interface that describe the physics.

The governing equations take the following form:

(A) Gas Phase

(1) continuity equation

$$\frac{\partial (u_g r)}{\partial x} + \frac{\partial (v_g r)}{\partial y} = 0 \qquad (1)$$

(2) momentum equation

$$u_g \frac{\partial u_g}{\partial x} + v_g \frac{\partial u_g}{\partial y} = -\frac{1}{\rho_g}\frac{\partial P}{\partial x} + \frac{\mu_g}{\rho_g}\frac{\partial^2 u_g}{\partial y^2} \qquad (2)$$

(3) boundary conditions

$$\begin{array}{ll} y = 0 & \left\{ v_g = 0;\; u_g = u_l(x);\; \mu_g \frac{\partial u_g}{\partial y} = -\mu_l \frac{\partial u_l}{\partial y} \right. \\ y = \infty & \left\{ u_g = U(x) \right. \end{array} \qquad (3)$$

(B) Liquid Phase

(1) continuity equation

$$\frac{\partial (u_l r)}{\partial x} + \frac{\partial (v_l r)}{\partial y} = 0 \qquad (4)$$

(2) momentum equation

$$u_l \frac{\partial u_l}{\partial x} + v_l \frac{\partial u_l}{\partial y} = \frac{\rho_g}{\rho_l} U \frac{dU}{dx} + \nu_l \frac{\partial^2 u_l}{\partial y^2} \qquad (5)$$

(3) boundary conditions

At $y = 0$, the same as gas phase

Because at the liauid-liquid interface, $y = -\infty$, several possible boundary conditions can be posed; two groups of them are written in following form.

Note that within the "Boundary-layer" assumptions, we assume that

$$\left.\frac{dp}{dx}\right)_{liq.} = \left.\frac{dp}{dx}\right)_{gas}$$

6.0 BOUNDARY CONDITIONS AT THE LIQUID-LIQUID INTERFACE

A simple sketch of this two phase boundary layer flow problem shows the different physical situations possible at liquid-liquid interface. Consider Figures (1) and (2) below.

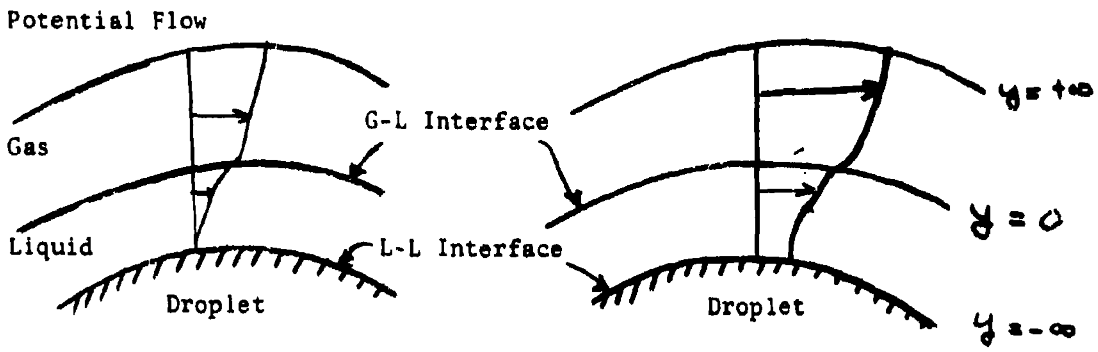


Figure 1 Figure 2

Figure (1) shows that the velocity is zero at liquid-liquid interface. Figure (2) depicts the case where the velocity is not zero there. As far as the physics is concerned at liquid-liquid interface, these are then really two different problems. The problem depicted in Figure (1) is a mass addition boundary layer flow problem, and the one in Figure (2) is called a "spallation" problem. Therefore, the two different boundary conditions to be specified at $y = -\infty$ are,

For problem (1), mass addition:	For problem (2), spallation:	
$u_\ell = 0$ $v_\ell \neq 0$	$u_\ell = K(x)\dfrac{\partial u_\ell}{\partial y}$ $v_\ell = 0$	(6)

where K above is a physical parameter which can be related to the surface tension or viscosity, both of which may be functions of the liquid temperature. As the first approximation we may assume K = constant.

The governing equations are just the classical boundary layer conservation equations in axisymmetric coordinates. In general, the standard non-dimensional similarity transformation variables will be useful, since they transform the two partial differential equations into one ordinary differential equation. Of course, we must check whether our boundary conditions can also be transformed in similarity space.

7.0 TRANSFORMATIONS

As is usual, we introduce the non-dimensional stream function ψ , so that the gas phase dependent variables are,

$$u = \frac{\ell}{r}\frac{\partial\psi}{\partial y} \qquad\qquad v = -\frac{\ell}{r}\frac{\partial\psi}{\partial x} \tag{7}$$

where ℓ is the characteristic length of the body.

Defining the similarity variables, ξ and η such that

$$\xi = \int_0^x \frac{r^2(x)\,U(x)}{\ell^2\,U_\infty}\,dx \tag{8}$$

and

$$\eta = \frac{U(x)\,r(x)\,y}{(2\nu_g\,U_\infty\,\xi)^{1/2}} \tag{9}$$

We arrive at the standard stream function, written as .

$$\psi = (2\xi U_\infty \nu_g)^{1/2}\,G(\eta) \tag{10}$$

This is therefore a definition of $G(\eta)$.

All the terms in momentum equation are transformed again in the standard way.

$$u = U(x)\,G'(\eta) \tag{11}$$

$$v = -\frac{r(x)\,U(x)\,(2U_\infty\nu_g)^{1/2}}{\ell\,U_\infty\,\xi^{1/2}}\left[\frac{G(\eta)}{2} + \left(\xi\frac{d(Ur)}{dx}\frac{\ell^2 U_\infty}{r^3 U^2} - \frac{1}{2}\right)\eta\,G'(\eta)\right] \tag{12}$$

$$\frac{\partial u}{\partial x} = G'(\eta)\frac{dU}{dx} + U\,G''(\eta)\left[\frac{\eta}{Ur}\frac{d(Ur)}{dx} - \frac{\eta\,U r^2}{2\,\ell^2\,\xi\,U_\infty}\right] \tag{13}$$

$$\frac{\partial u}{\partial y} = \frac{U^{2} r \; G''(\eta)}{\ell (2 \nu_g U_\infty \xi)^{1/2}} \qquad (14)$$

$$\nu_g \frac{\partial^2 u}{\partial y^2} = \frac{U^3 r^2 \; G'''(\eta)}{2 \ell^2 U_\infty \xi} \qquad (15)$$

7.1 Gas Phase

Substituting all these terms into gas phase momentum equations, an ordinary differential equation is obtained as

$$G''' + G G'' + \beta(\xi)(1 - G'^2) = 0 \qquad (16)$$

where

$$\beta(\xi) = 2\xi \frac{d \log U(x)}{d \xi} \qquad (17)$$

For example at the point $\pi/4$ from the leading edge, $\beta(\xi) = 0.295$. (See Appendix I.)

The boundary conditions are now.

$$\begin{aligned} \eta = 0 \quad & \left\{ G = 0 \quad \text{and} \quad G'(0) = \tilde{U}(x) \right\} \\ \eta = \infty \quad & \left\{ G'(\infty) = 1.0 \right. \end{aligned} \qquad (18)$$

Note that, in general, at $\eta = 0$, $G'(0) \neq 0$.

7.2 Liquid Phase

Similarly, the non-dimensional stream function and the similarity variables for liquid phase are defined as

$$\psi = (2\xi U_\infty \nu_l)^{1/2} L(\eta) \tag{19}$$

$$\xi = \int_0^x \frac{r^2(x)\, U(x)\, dx}{l^2 U_\infty} \tag{20}$$

$$\eta = \frac{r(x)\, U(x)\, y}{l\sqrt{2\nu_l U_\infty \xi}} \tag{21}$$

The only difference between the above quantities equation (19-21) for the liquid and those of the gas is the kinematic viscosity coefficient ν_l in equations (8-10). Again a similar procedure is used to transform to the similarity space. The governing differential equation is obtained for the liquid phase as,

$$L''' + LL'' + \beta(\xi)\left[\frac{\rho_g}{\rho_l} - L'^2\right] = 0 \tag{22}$$

Notice the term ρ_g/ρ_l in the above equation. The operator is therefore different than that of equation (16) for $G(\eta)$. The boundary conditions for equation (22) are as follows. For

(1) The mass addition problem:

$$\eta = 0 \left\{ L(0) = 0;\ L'(0) = G'(0);\ L''(0) = G''(0)\Gamma \right. \tag{23}$$

$$\eta = -\infty \left\{ L'(-\infty) = 0 \right. \tag{24}$$

(2) The spallation problem:

$$\eta = 0 \left\{ L(0) = 0;\ L'(0) = G'(0);\ L''(0) = G''(0)\Gamma \right. \tag{25}$$

$$\eta = -\infty \quad \left\{ L'(-\infty) = k\, L''(-\infty) \right\} \tag{26}$$

where k is the non-dimensional form of the parameter K.

By examination of both the governing equations and boundary conditions, it is easy to see that this two-phase boundary-layer flow problem is coupled by the interface conditions. (See equations (23) and (18)). Because our main goal in this analysis is to find how much liquid has been stripped away from the liquid droplet, the liquid-liquid interface boundary condition is really the most important part in this analysis. If the boundary condition at $\eta = -\infty$ is defined, the subsequent interest is the asymptotic solution to equation (22) for the liquid phase. The specification of our imposed boundary conditions there will depend upon the behavior of the asymptotic solution of the problem. In equation (23) or (25), we have attempted to impose at $\eta = -\infty$ either $L' = 0$ or $L' = k\, L''(-\infty)$; the asymptotic solutions will provide a check of whether any or either of these are allowed.

NOTE: $\Gamma = \dfrac{\rho_g \sqrt{\nu_g}}{\rho_\ell \sqrt{\nu_\ell}}$ and is typically less than 0.1.

8.0 THE PROCEDURE FOR FINDING THE ASYMPTOTIC SOLUTION

One boundary condition we plan to set at the liquid-liquid interface is that $L'(-\infty)=0$. This implies that $L(-\infty)\simeq C,\ C>0$. Following the usual procedure to find the asymptotic solution of the governing equation, namely

$$L''' + LL'' + \beta(\xi)\left[\beta_3/\beta - L'^2\right] = 0 \qquad (22)$$

we first differentiate the above and obtain

$$L^{IV} + LL''' + L'L''(1-2\beta)=0 \qquad (27)$$

Applying the conditions $L'(-\infty)=0$ and $L(-\infty)=C$ to the above relation we see that

$$L^{IV} + CL''' = 0 \qquad (28)$$

Integrating the above once, we have

$$L''' = e^{-C\eta} + B_1 \qquad (29)$$

Now since $L'''(-\infty)$ must be zero due to breaking up condition from the droplet at $-\infty$, this implies that $B_1 = 0$ in the above relation.

Integrating equation (29) once more, then

$$L'' = -\frac{1}{C}e^{-C\eta} + B_2 \qquad (30)$$

Also $L''(-\infty)$ must be zero (since $L'(-\infty) = 0$); this implies that $B_2 = 0$ in equation (30). The proceeding boundary condition means no shearing stress at $\eta = -\infty$, which means that actually some fluid has broken away from the liquid droplet

Continuing, we integrate once more, and find that

$$L' = \frac{1}{C^2} e^{-C\eta} + B_3 \tag{31}$$

Again $L'(-\infty) = 0$, the original supposition. Therefore, $B_3 = 0$. Finally the solution for $L(\eta)$ is

$$L = -\frac{1}{C^3} e^{-C\eta} + B_4 \tag{32}$$

We note that since $L(-\infty) \cong C$, then $B_4 = C$. thus

$$L = -\frac{1}{C^3} e^{-C\eta} + C \tag{33}$$

L is the asymptotic solution for the mass addition problem

In the process of finding the asymptotic solution, one can make a very important observation as to the physical phenomenon occurring in the two boundary layer problem. That is, L'' can be expressed as,

$$L'' = -\frac{1}{C} e^{-C\eta} \tag{34}$$

This expression indicates that there is an inflection point for the velocity distribution of the liquid phase. The following

figure gives an indication of the resulting expected velocity distribution that must result; if we are to match the boundary conditions at y = 0.

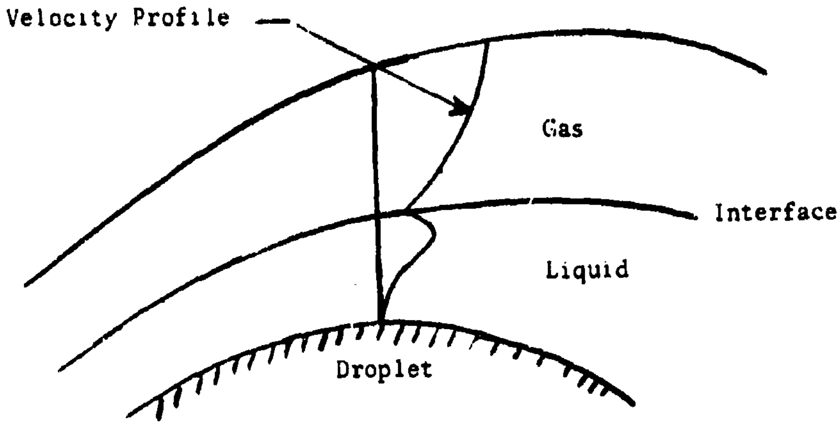


Figure 3

The physics should be carefully studied and investigated, and we plan to do so. A suitable explanation for the maximum velocity in the liquid layer is necessary.

The other boundary condition for the spallation problem we plan to specify is $L'(-\infty) = k L''(-\infty)$. This implies that

$$L'(-\infty) = e^{-\eta/k} + C \tag{35}$$

The above expression indicates that $L'(\infty) \neq 0$. As η approaches $-\infty$, $L'(-\infty)$ approaches C. This corresponds to the velocity distribution in Figure 2. Because this integration constant C is a function of many physical properties, it is difficult to determine it a-priori. For this reason, a further

study of the asymptotic solution of the, spallation problem is required but is not made here. We plan to attempt such an analysis later.

9.0 PROCEDURAL OUTLINE FOR SOLVING THE COUPLED EQUATIONS

The equation of motion in the gas and liquid phase have been transformed into ordinary non-linear differential equations. Because these equations are non-linear and the boundary conditions are given at more than one point (split), the existence of a solution to this problem is not guaranteed. Therefore, at this point, an analytic approach which attempts to solve this problem is deferred until later.

A numerical solution is, therefore, applied to this problem. Because the boundary conditions at the gas-liquid interface are indefinite, the procedure used in solving this problem is to first guess a condition there. Then an iteration process is applied until all the other boundary conditions are satisfied. For example, at y = 0, we guess the slipping velocity, that is a value for G'(0), to meet the condition that $G'(\infty) = 1$. A part of the solution is then a value for G''(0).

We then proceed to the liquid phase; we start here by setting $L'(0) = G'(0)$, and $L''(0) = G''(0)\,\Gamma$. Again $L(0) = 0$. We solve numerically the governing Falkner-Skan equation for $L(\eta)$, and if we are doing the mass addition problem, we stipulate that at $\eta = -\infty$, $L'(-\infty) = 0$. If we cannot satisfy this condition at the liquid-liquid interface, then we go all the way back to the gas phase, guess a new value for G'(0), obtain a new value for G''(0), and then solve the liquid phase with these new conditions. Eventually (recall all these are one value of $\beta(\xi)$) there will be only one slipping velocity $G'(0) = L'(0)$ which

will satisfy both outer boundary conditions, which are $G'(\infty) = 1$ and $L'(-\infty) = 0$. We then proceed to other points along the streamline, where we vary β from the near the stagnation point to the maximum (shoulder location) height at $x/R = \pi/2$.

Mass Stripped:

We can easily determine the mass added to the liquid boundary layer at one β location by the relation,

$$\frac{dm}{dt} = \pi D \rho_\ell \int_0^\infty u_\ell \, dy$$

$$= \pi D \rho_\ell \, \ell \, \frac{\sqrt{2 \nu_\ell U_\infty \xi}}{r(x)} \int_0^\infty L'(\eta) \, d\eta \tag{36}$$

Thus the total mass stripped in an arbitrary time period, T, is

$$m = \pi D \rho_\ell T \, \frac{\ell \sqrt{2 \nu_\ell U_\infty \xi}}{r(x)} \int_0^\infty L'(\eta) \, d\eta \tag{37}$$

10.0 COMPUTER PROGRAM (SEE APPENDIX II)

Before a numerical solution of the Falkner-Skan equation can be made, it must be reduced from a split boundary value problem to an initial value problem. The boundary conditions for the generalized Falkner-Skan equation usually prescribe $f(0)$, $f'(0)$, $f''(\infty)$ To numerically integrate the equation, f''(0) must be known. Note that $f(\eta)$ corresponds to either $G(\eta)$ or $L(\eta)$.

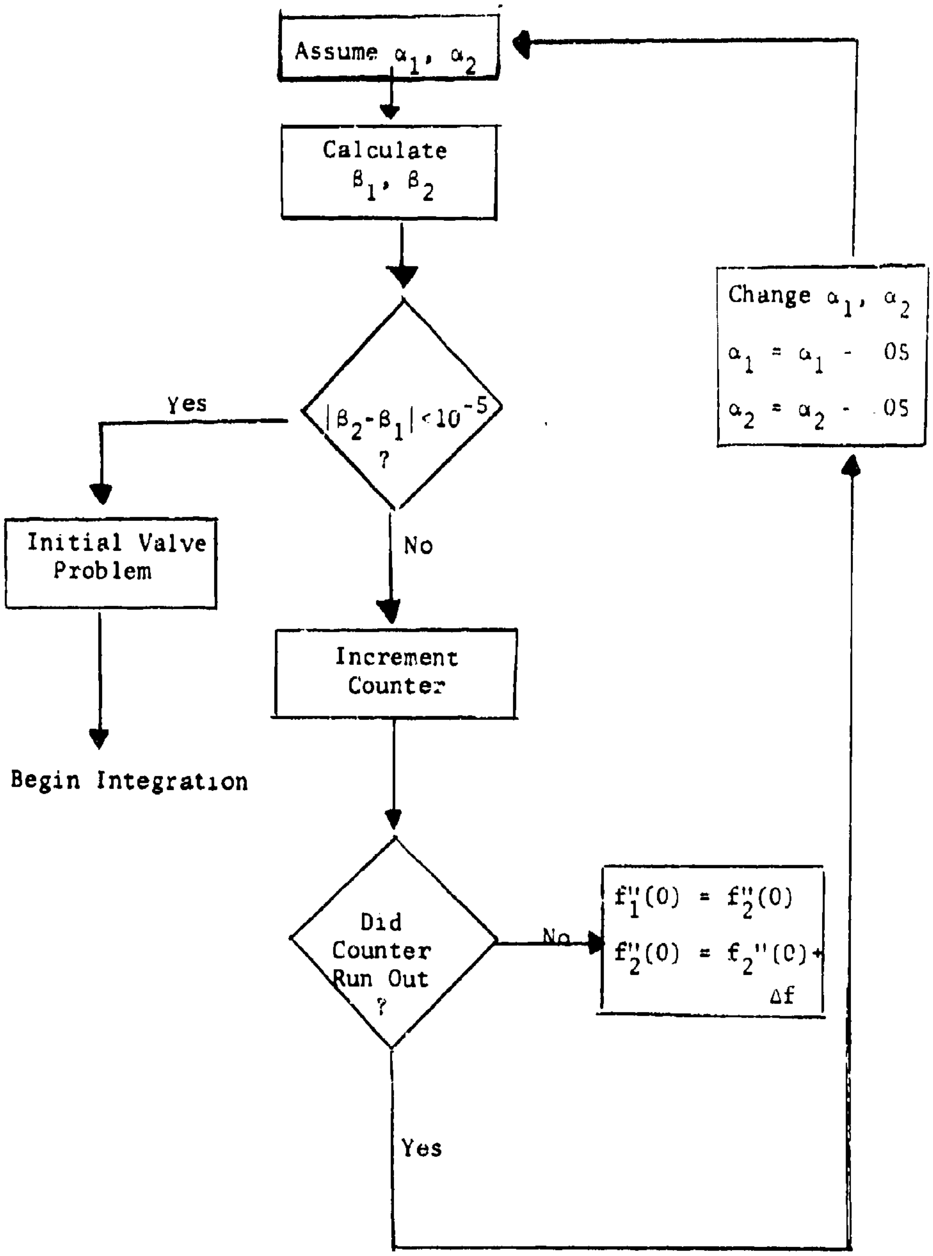
To solve for f''(0), we must use the other boundary conditions, and a 4th order Runge-Kutta method. The following scheme is used.

$$\Delta f''(0) = \frac{\alpha_2 - \alpha_1}{\bar{\beta}_2 - \bar{\beta}_1}\left[f'(\infty) - \bar{\beta}_2\right]$$

where α_1, α_2 are initial guesses for $f_1''(0)$ and $f_2''(0)$, $\bar{\beta}_1$, $\bar{\beta}_2$ are the corresponding values of $f_1'(\infty)$ and $f_2'(\infty)$. The $\Delta f''(0)$ so obtained is added to $f''(0)$, and $f_1''(0)$ becomes $f''(0)$. This process is repeated until $|f'(\infty) - f_2'(\infty)| < 10^{-5}$. The value of f''(0) which satisfies this condition is then the correct value. This method, however, is not full-proof. This entire process depends on the initial guesses of f''(0). This initial guess must be within 50% of the true value, or the method will not converge.

Once the method converges, a least squares curve fit is used to find the relationship between f'(0) and f''(0). The values obtained are carved out for a three degree polonomial. This should be more than sufficient, since values of f'(0) are usually less than .5. The least squares curve fit was based on a weighting function of 1 for all values.

FLOW CHART



Assume α_1, α_2
Calculate β_1, β_2
$|\beta_2 - \beta_1| < 10^{-5}$?
Yes
Initial Valve Problem
Begin Integration
No
Increment Counter
Did Counter Run Out ?
No
$f_1''(0) = f_2''(0)$
$f_2''(0) = f_2''(0) + \Delta f$
Yes
Change α_1, α_2
$\alpha_1 = \alpha_1 - .05$
$\alpha_2 = \alpha_2 - .05$

$\theta = 30°$, $\beta = .1850$

f' (0)	f" (0)
.01	.6698
.02	.6668
.03	.6638
.04	.6607
.05	.6575
.06	.6542
.07	.6508
.08	.6472
.09	.6436
1.0	.6399
1.2	.6322
1.4	.6241
1.6	.6157
1.8	.6069
2.0	.5978
2.5	.5735
3.0	.5471
3.5	.5188
4.0	.4885
5.0	.4228

$A_o = .67267$, $A_1 = -0.2777$, $A_2 = -0.50645$, $A_3 = 0.12543$

$\theta = 45°$, $\beta = .2950$

f' (0)	f''(0)
.01	.7666
.02	.7627
.03	.7586
.04	.7544
.05	.7502
.06	.7458
.07	.7413
.08	.7368
.09	.7327
1.0	.7274
1.2	.7177
1.4	.7076
1.6	.6972
1.8	.6864
2.0	.6753
2.5	.6460
3.0	.6148
3.5	.5816
4.0	.5466
5.0	.4713

$A_o = .77061$, $A_1 = -0.38473$, $A_2 = -0.47954$, $A_3 = 0.10396$

$\theta = 60°$, $\beta = 0.0808$

f' (0)	f"(0)
01	.5646
.02	.5630
.03	.5613
.04	.5594
.05	.5575
.06	.5554
.07	.5532
.08	.5509
.09	.5485
1.0	.5460
1.2	.5407
1.4	.5349
1.6	.5288
1.8	.5223
2.0	.5154
2.5	.4966
3.0	.4756
3.5	.4526
4.0	.4276
5.0	.3721

$A_o = .56626$ $A_1 = -0.14753$, $A_2 = -0.565176$, $A_3 = .167826$

$\theta = 90°, \beta = 0.0$

$f'(0)$	$f''(0)$
.01	.4695
.02	.4693
.03	.4689
.04	.4684
.05	.4677
.06	.4669
.07	.4660
.08	.4650
.09	.4638
1.0	.4625
1.2	.4595
1.4	.4561
1.6	.4522
1.8	.4479
2.0	.4431
2.5	.4295
3.0	.4135
3.5	.3953
4.0	.3750
5.0	.3287

$A_o = .46984$, $A_1 = -0.0095135$, $A_2 = -0.66554$, $A_3 = .24078$

11.0 CONCLUSION

This report has dealt with the problem of the interaction between a liquid droplet and the convective flow field surrounding it. This problem has received some attention by other investigators, as indicated in the list of references. Most studies have been experimental ones, and one in particular, by Nicholls and Ranger, has attempted an approximate boundary-layer analysis by first assuming arbitrary simple velocity distributions, both in the gas and liquid layers.

This report is an attempt to solve the problem of aerodynamic stripping of liquid from a spherical drop by solving (numerically) the full coupled steady boundary layer equations (for incompressible flow) and stipulating certain realistic interface conditions. No solutions are given since the work represents a funded effort of only one half year. However, considerable insight into the nature of the coupling behavior of the gas and liquid viscous flows has been made, and we expect later to carry out to completion the solution to the model proposed. The main parameter of interest to be solved for will be mass of fluid in the circumferential liquid layer that was swept along by the gas stream.

12.C ACKNOWLEDGMENTS

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APPENDIX I

CALCULATION OF THE VALUES FOR β

$$\beta(\xi) = 2\xi U_\infty \frac{\ell^2}{r^2} \frac{dU_1}{dx} \frac{1}{U_1^2}$$

$$\begin{cases} U_1 = \frac{3}{2} U_\infty \sin \frac{X}{R} & , \quad U_1^2 = \frac{9}{4} U_\infty^2 \sin^2 \frac{X}{R} \\ r = R \sin \frac{X}{R} & ; \quad r^2 = R^2 \sin^2 \frac{X}{R} \end{cases}$$

Thus

$$\frac{dU_1}{dX} = \frac{3}{2} U_\infty \frac{d \sin X/R}{dX} = \frac{3}{2} U_\infty / R \cdot \cos \frac{X}{R}$$

Now

$$\xi = \int_0^X \frac{R^2 \sin^2 \frac{X}{R} \cdot \frac{3}{2} U_\infty \sin \frac{X}{R}}{\ell^2 U_\infty} dX$$

$$= \int_0^X \frac{R^2 \sin^3 \frac{X}{R}}{\ell^2} dX$$

$$= \int_0^X \frac{R^3}{\ell^2} \sin^3 \frac{X}{R}\, d\left(\frac{X}{R}\right)$$

$$= \frac{R^3}{\ell^2} \left[\frac{2}{3} + \frac{\cos^3 \frac{X}{R}}{3} - \cos \frac{X}{R} \right]$$

Then

at $\frac{x}{R} = \frac{\pi}{2}$, $\quad \xi = \frac{2}{3}\frac{R^3}{\ell^2}$

$\frac{\pi}{4}$, $\quad \xi = \left(\frac{8-3\sqrt{2}}{12}\right)\frac{R^3}{\ell^2}$

$\frac{\pi}{3}$, $\quad \xi = \frac{5}{24}\frac{R^3}{\ell^2}$

$\frac{\pi}{6}$, $\quad \xi = \frac{(16-9\sqrt{3})}{24}\frac{R^3}{\ell^2}$

Finally

$$\beta(\xi) = 2\xi U_0 \frac{\ell^2}{r^2}\left(\frac{3}{2}\frac{U_0}{R}\cos\frac{x}{R}\right)\frac{4}{9U_0^2\sin^2\frac{x}{R}}$$

$$= \frac{4}{3}\xi\,\frac{\cos\frac{x}{R}}{\sin^2\frac{x}{R}}\,\frac{\ell^2}{r^2 R}$$

Thus

At $\frac{\pi}{2}$: $\quad \beta(\xi) = 0$

$\frac{\pi}{4}$: $\quad \beta(\xi) = \frac{4}{3}\left(\frac{8-3\sqrt{2}}{12}\right)\frac{R^3}{\ell^2}\cdot\frac{\ell^2}{r^2R}\frac{1}{\sqrt{2}} \quad 0.291$

$\frac{\pi}{3}$: $\quad \beta(\xi) = 0.185$

$\frac{\pi}{6}$: $\quad \beta(\xi) = 0.081$

APPENDIX II

COMPUTER PROGRAM FOR SOLVING THE FALKNER-SKAN EQUATION

```
      IMPLICIT REAL*8(A-H,O-Y)
      DIMENSION Y1(200),Y2(200),Y3(200),DY1(200),DY2(200),DY3(200),XINC(
     1200),ALPHA(10),BETAA(10),TRY(10),THETA2(100),THETA1(100),PRA(20),Z
     1TX(2),ZTY(2),ZF1(200),ZF2(200),ZF3(200),ZF4(200),ZXINC(200)
      CALL UNDERZ('OFF')
      CALL ERRSET(207,256,-1,1)
C
C
C
C              INITIALIZE VARIABLES
C
C
C
      READ(5,10)(PRA(J),J=1,20)
   10 FORMAT(20F4.2)
      H=5.0D-2
      DNSTY=1.D-1
      DNSTY=1.0D0
      TRY(1)=0.0D0
      TRY(2)=0.295D0
      TRY(3)=0.0808D0
      TRY(4)=0.185D0
      ZF2(1)=90.
      ZF2(2)=45.
      ZF2(3)=60.
      ZF2(4)=30.
      X=0.0D0
      M=101
      BETA=1.0D0
      DO 5000 JKL=1,4
      BETA=TRY(JKL)
      WRITE(6,506) ZF2(JKL),TRY(JKL)
  506 FORMAT('1',23X,'THETA = ',F5.1,' DEGREES, BETA = ',F7.4,/,/)
      WRITE(6,507)
  507 FORMAT(' ',23X,'F''(0)     ',10X,'F'''(0)',/)
      DO 4999 LM=1,20
C
C
C
C        NOTE:  THE VALUE OF CONST IS TO BE USED AS FOLLOWS:************
C
C               IF BETA=0.0    THEN CONST=1.0 OR 0.5 DEPENDING ON THE FORM
C               OF THE BLASIUS EQUATION TO BE USED
C
C               IF BETA .NE. 0.0    THEN CONST=1.0  ALWAYS!!!!!
C
C
C
      CONST=5.0D-1
      CONST=1.0D0
```

```
      FINFKN=0.0D0
      FINFKN=1.0D0
      Y1(1)=0.0D0
      Y2(1)=0.01D0
      Y2(1)=PRA(LM)
      ZF1(LM)=PRA(LM)
      ALPHA(1)=0.5D0
      ALPHA(2)=0.6D0
      INCREM=0
      ICOUNT=0
C
C
C
C               ************ MAIN      PROGRAM **************
C
C
C
      HOLD1=ALPHA(1)
      HOLD2=ALPHA(2)
4524  DO 610 LL=1,262
      IF(INCREM.EQ.2) GO TO 6314
      GO TO 6315
 6314 ALPHA(1)=HOLD1+3.0D-2
      ALPHA(2)=HOLD2+3.0D-2
      HOLD1=ALPHA(1)
      HOLD2=ALPHA(2)
      INCREM=0
      ICOUNT=0
6315  DO 150 JJ=1,2
      Y3(1)=ALPHA(JJ)
      KKK=JJ
      XINC(1)=X
      DY1(1)=Y2(1)
      DY2(1)=Y3(1)
      DY3(1)=-CONST*Y1(1)*Y3(1)-(BETA*(DNSTY-(Y2(1)**2)))
      DO 1 J=2,M
      KN=J
      P1=H*DY1(J-1)
      P2=H*DY2(J-1)
      P3=H*DY3(J-1)
      A=Y1(J-1)+(5.0D-1)*P1
      B=Y2(J-1)+(5.0D-1)*P2
      C=Y3(J-1)+(5.0D-1)*P3
      Q1=H*B
      Q2=H*C
      Q3=H*((-CONST)*A*C-(BETA*(DNSTY-(B**2))))
      A=Y1(J-1)+(5.0D-1)*Q1
      B=Y2(J-1)+(5.0D-1)*Q2
      C=Y3(J-1)+(5.0D-1)*Q3
      R1=H*B
      R2=H*C
      R3=H*((-CONST)*A*C-(BETA*(DNSTY-(B**2))))
      A=Y1(J-1)+R1
      B=Y2(J-1)+R2
      C=Y3(J-1)+R3
      S1=H*B
      S2=H*C
```

```
      S3=H*((-CONST)*A*C-(BETA*(DNSTY-(B**2))))
      Y1(J)=Y1(J-1)+(P1+(2.0D0)*Q1+(2.0D0)*R1+S1)/6.0D0
      Y2(J)=Y2(J-1)+(P2+(2.0D0)*Q2+(2.0D0)*R2+S2)/6.0D0
      Y3(J)=Y3(J-1)+(P3+(2.0D0)*Q3+(2.0D0)*R3+S3)/6.0D0
      DY1(J)=Y2(J)
      DY2(J)=Y3(J)
      DY3(J)=-CONST*Y1(J)*Y3(J)-(BETA*(DNSTY-(Y2(J)**2)))
      IF(Y2(J).GT.1.5D0) GO TO 3000
      XINC(J)=XINC(J-1)+H
1     CONTINUE
3000  IF(KKK.EQ.2) GO TO 390
      FINF1=Y2(KN)
      GO TO 150
390   FINF2=Y2(KN)
150   CONTINUE
      XB=FINFKN-FINF2
      XC=(ALPHA(2)-ALPHA(1))/(FINF2-FINF1)
      DELTF=XC*XB
      ALPHA(1)=ALPHA(2)
      ALPHA(2)=ALPHA(2)+DELTF
      IF(DABS(FINFKN-FINF2).LE.1.0D-5) GO TO 505
      ICOUNT=ICOUNT+1
      IF(ICOUNT.EQ.10) INCREM=2
610   CONTINUE
C
C
C
C PRINT OUT RESULTS
C
C
C
 505  WRITE(6,504) Y2(1),Y3(1)
 504  FORMAT(' ',23X,F14.10,5X,F14.10)
4999  CONTINUE
      ICOUNT=0
      INCREM=0
5000  CONTINUE
      STOP
      END
/*
//GO.SYSIN DD *
 .01 .02 .03 .04 .05 .06 .07 .08 .09 .10 .12 .14 .16 .18 .20 .25 .30 .35 .4
/*
```